Pathria 1-15

$\frac{10}{10}$

We begin by noting that $E$, $C_v$, and $C_p$ can all be expressed in terms of the # of degrees of freedom an ideal gas has. dofs: $\delta$.

$$E = \frac{\delta}{2} NkT \quad , \quad C_v = \frac{\delta}{2} Nk \quad , \quad C_p = \frac{\delta+2}{2} Nk$$

Thus, $\gamma = \frac{c_p}{c_v} = \frac{\delta+2}{\delta} \Rightarrow \delta = \frac{2}{\gamma - 1}$

Now, we are given two ideal gasses:

gas 1: $E_1 = \frac{\delta_1}{2} N_1 k T_1$ and $\delta_1 = \frac{2}{\gamma_1 - 1}$

gas 2: $E_2 = \frac{\delta_2}{2} N_2 k T_2$ and $\delta_2 = \frac{2}{\gamma_2 - 1}$

When mixed, $E = E_1 + E_2$

$$\frac{\delta}{2} NkT = \frac{\delta_1}{2} N_1 k T_1 + \frac{\delta_2}{2} N_2 k T_2$$

$$\frac{2NkT}{\gamma - 1} = \frac{2N_1 k T_1}{\gamma_1 - 1} + \frac{2N_2 k T_2}{\gamma_2 - 1}$$

But the mixture will equilibrate at $T = T_1 = T_2$, so we get cancelation

$$\frac{N}{\gamma - 1} = \frac{N_1}{\gamma_1 - 1} + \frac{N_2}{\gamma_2 - 1}$$

Finally, note that $f_1 = N_1/N$ and $f_2 = N_2/N$

$$\therefore \boxed{\frac{1}{\gamma_1 - 1} = \frac{f_1}{\gamma_1 - 1} + \frac{f_2}{\gamma_2 - 1}}$$